Impact of noise on domain growth in electroconvection

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The growth and ordering of striped domains has recently received renewed attention due in part to experimental studies in diblock copolymers and electroconvection. One surprising result has been the relatively slow dynamics associated with the growth of striped domains. One potential source of the slow dynamics is the pinning of defects in the periodic potential of the stripes. Of interest is whether or not external noise will have a significant impact on the domain ordering, perhaps by reducing the pinning and increasing the rate of ordering. In contrast, we present experiments using electroconvection in which we show that a particular type of external noise decreases the rate of domain ordering.

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The study of domain growth after a sudden change to a system (or a quench) has implications for a wide range of applications and fields, from processing binary mixtures of fluids to the organization of crystalline domains in a solid [1]. Domain growth occurs when different spatial regions of a system are in different states, and the size of these regions changes with time. A common method to generate such a situation is to take a uniform system and subject it to a sudden change in external parameters, such that the system can now exist in two or more degenerate states. Regions form that select from the possible states, creating an inhomogeneous system of domains that proceed to grow. This process is often referred to as domain ordering or coarsening. Our understanding of domain ordering focuses on the case when the system is spatially uniform within each domain. In this case, one can generally understand the coarsening by considering the motion of the topological defects in the system [1]. The situation is less clear when the domains themselves contain spatial structure. In this regard, domains of stripes have received significant attention [2–13].

Stripes, or more generally patterns, occur in a wide range of systems [14,15], including convecting fluids, animal coats, polymer melts, and ferromagnets. Stripes occur both as an equilibrium state of the system, such as in diblock copolymers, and as a result of external driving, as in convection in fluids. Theoretical studies of the ordering of striped domains [2–4,6,8,9,11–13] have focused on studies of model equations, such as the Swift-Hohenberg equation. In general, simulations find that the growth of striped domains occurs slower than might be expected from our knowledge of the growth of uniform systems. For sufficiently late times, it is postulated that the characteristic size of domains in these systems scales as a power of time, with a characteristic exponent known as the growth exponent. For uniform domains in systems approaching an equilibrium state, it is known that the exponents are $1/2$ if the order parameter is not conserved and $1/3$ for a conserved order parameter [1]. For striped systems, growth exponents of various values are reported, including $1/3$, $1/4$, and $1/5$. There is evidence that for sufficiently large quenches, the system becomes glassy, and scaling breaks down [6,13]. Of particular interest to the work in this paper are simulations that focus on anisotropic systems [13] and recent simulations that focus on the impact of noise on the coarsening of stripes [11,12]. This will be discussed in more detail later.

The ordering of striped domains has been studied experimentally both for the diblock copolymer case [5,7] and the electroconvection in a nematic liquid crystal [10,16,17]. The work with diblock copolymers strongly suggests that topological defects play a central role in the ordering of striped domains [7]. The work in electroconvection is interesting because the system is an example of coarsening in an anisotropic, driven system. The work in this paper focuses on domain coarsening in this system.

Nematic liquid crystals are fluids in which the molecules exhibit long-range orientational order [18]. The average axis along which the molecules are aligned is referred to as the director. By proper treatment of the boundaries, samples with uniform director alignment can be prepared. When an electric voltage is applied to such a sample, there is a critical voltage at which convective flow of the fluid occurs. This phenomenon is known as electroconvection [19–21]. There is a corresponding periodic variation of the director field. When the axis of the convection rolls forms a nonzero angle with the undistorted director field, the pattern is referred to as oblique rolls. This is a degenerate state because patterns with an angle θ (zig rolls) and $-\theta$ (zag rolls) are degenerate. By applying a sudden increase in voltage from below to above the critical value, a pattern of zig and zag domains is created. This pattern coarsens, or orders, in time. The ordering of the system is consistent with power-law growth in time [10], but the growth is anisotropic, occurring at different rates parallel and perpendicular to the director [16].

Because electroconvection is driven electrically, it is a relatively easy system for studies of the impact of noise. Simulations of an anisotropic Swift-Hohenberg model [13] suggest that the pinning of defects is relevant to the coarsening of the domains in electroconvection. Simulations for isotropic systems suggest that noise can alter the dynamics of the domain growth by changing the potential in which the defects move, effectively acting as a thermal bath [11,12]. Though these simulations were performed for isotropic systems, there is no obvious reason that a similar phenomenon would not occur in electroconvection.

In considering the impact of noise on the domain growth, one can imagine different types of noise. The two classic cases are additive noise and multiplicative noise. These are

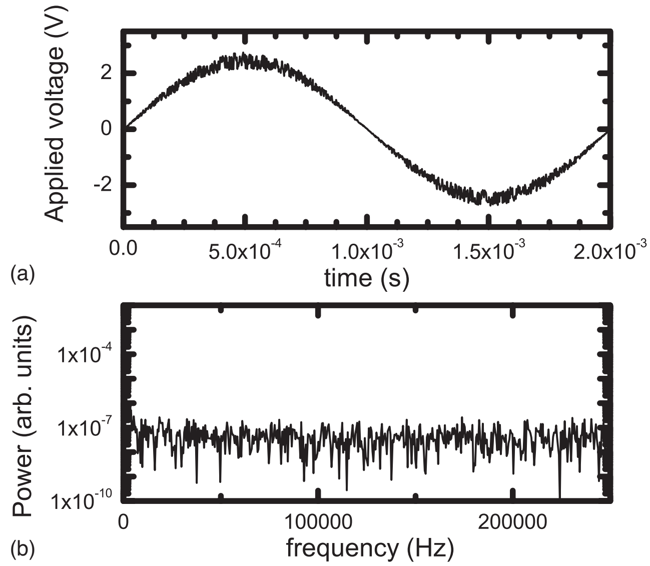

FIG. 1. (a) A single cycle of a typical waveform that is used to drive electroconvection. A noise amplitude of 0.6 V is used so that it is visible to the reader. (b) Power spectrum for the signal plotted in (a).

best defined in the context of an amplitude equation formulation (or envelope equation) in which the fast variation (the period corresponding to the fundamental pattern) is removed and only long wavelength changes in the pattern are studied. In this formalism, additive noise is described by the addition of a noise term to the equation that is not directly coupled with the amplitude [2,11]. Multiplicative noise is the addition of a term that consists of a noise factor multiplying the amplitude [12]. Since it is standard to have the driving represented by an appropriate dimensionless parameter times the amplitude, multiplicative noise enters the equations as an additive factor to the driving term. A surprising feature of the theoretical studies is the finding that the rate of domain growth increases in the presence of noise [2,11,12]. (For sufficiently large noise strengths, the pattern ultimately breaks up into random domains [2,11,12].) Since additive and multiplicative noise sources have the same impact, it is possible that any generic noise would increase the rate of coarsening. It should be noted that a common feature of these simulations is the use of spatially and temporally random noise, independent of whether or not the noise was additive or multiplicative. To test one aspect of the generality of these results, we focus on spatially uniform temporal noise in the experiments described here.

The details of the experimental apparatus are described in Ref. [22]. The nematic liquid crystal N4 was doped with 0.1 wt % of tetra n-butylammonium bromide [$(C_4H_9)4NBr$]. We previously characterized this system [23]. The conductivity is on the order of $10^{-6}\ \Omega^{-1}\,m^{-1}$, and the cutoff frequency is greater than 10 000 Hz. Commercial cells [24] with a quoted thickness of 25 μm and 1 cm^2 electrodes were used, giving an aspect ratio of 400. The cells were treated so that the director has a uniform planar alignment (parallel to the top and bottom of the plates). The direction of the undistorted director is taken as the x axis and the direction perpendicular to the plates is taken as the z axis. The y axis is chosen to form a standard right-handed coordinate system. The average wavelength of the rolls was 51 μm. The sample temperature was maintained at $35.0\pm0.002\ ^\circ$C.

Typically in studies of electroconvection, an ac voltage of the form $V(t)=V_o\cos(\omega t)$ is applied perpendicular to the cell, where V_o is the amplitude of the applied voltage and $\omega/2\pi$ is the driving frequency. A number of experiments have considered different types of noise and its impact on various transitions in electroconvection [25–27]. For all of the experiments reported here, we have selected $V(t)=[V_o+\xi(t)]\cos(\omega t)$, where $\xi(t)$ is a random noise term chosen with a uniform probability from the range $-\xi_m\leqslant\xi(t)\leqslant\xi_m$. Figure 1 shows sample waveforms with their corresponding power spectra with $\xi_m=0.6$ V. The basically flat power spectra indicate the randomness of the noise. This was achieved by using a built-in pseudorandom number generator. The seed value was changed often by using the time (in seconds) elapsed since New Year 1970 as the seed. After one cycle of the waveform is randomized, that cycle is then repeated to create a periodic random waveform.

It should be noted that the noise we add is similar to the multiplicative noise studied theoretically. However, we add the noise to the raw voltage and the control parameter is the square of the voltage. Second, because of experimental limitations, the noise has a periodic element as only a single cycle is generated randomly. Finally, as mentioned, we use spatially uniform noise. Therefore, these experiments are not aimed at testing a specific class of noise, such as additive or multiplicative. Instead, the goal is to test the general observation from simulations that various classes of noise increase the rate of coarsening.

The dimensionless parameter $\epsilon=(V/V_c)^2-1$ characterizes the depth of the quench. Here $V_c\approx5.4$ V is the onset voltage for electroconvection at the applied frequency $(f=\omega/2\pi)$ of interest. For all of the experiments reported here, $f=500$ Hz. The introduction of noise did have a small effect on the onset voltage, as shown in Fig. 2. It should be noted that previous work on the impact of noise on the onset volt-

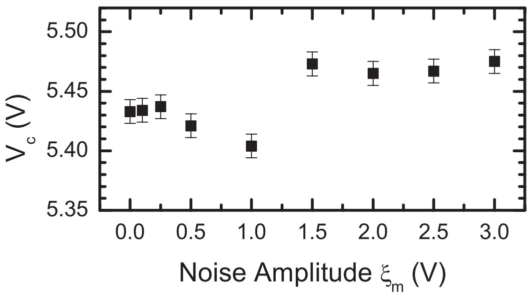

FIG. 2. Plot of the critical voltage for the onset of electroconvection as a function of the noise amplitude. Note the transition is at a noise amplitude of approximately 1.5 V.

age indicates that the onset voltage is independent of the noise strength for noise strengths below approximately 5 V [25]. Therefore, the weak dependence of the onset voltage on the noise strength is not surprising. In order to study domain coarsening, voltage quenches were used that started from a uniform state and suddenly applied sufficient voltage to reach $\epsilon=0.20\pm0.02$, where ϵ is computed using V_c and V_o in the absence of noise. This represents a quench to a region roughly in the middle of the oblique roll regime [23]. The quoted error represents the maximum variation in ϵ due to *both* the variation in V_c and the application of the noise term $\xi(t)$ to the signal. [Note: because of the form of the noise, the impact of $\xi(t)$ on ϵ is small.] We define a relative noise strength $\eta=\xi_m/V_c$, where V_c is again taken as the critical voltage in the absence of noise.

After the application of the quench, domains of zig and zag, separated by grain boundaries and vertical walls of dis-

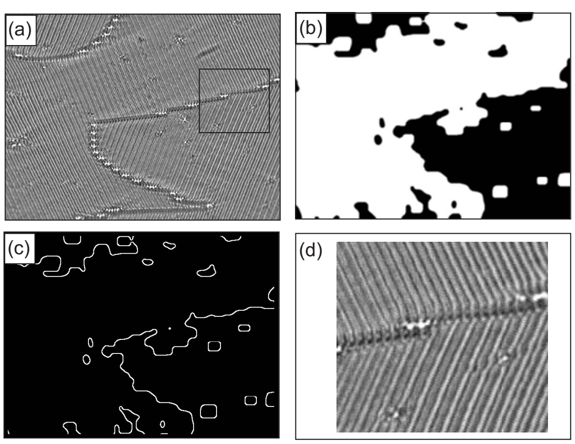

FIG. 3. Three images illustrating the process by which domains are defined. The images are all 2.03 mm × 1.41 mm. (a) Raw image of electroconvection showing two regions of zag rolls in between two regions of zig rolls. (b) The processed image in which zag is white and zig is black. (c) The extracted domain walls from image (b). (d) Shows the region of (a) indicated by the black box magnified by a factor of 3. This highlights the zig and zag rolls.

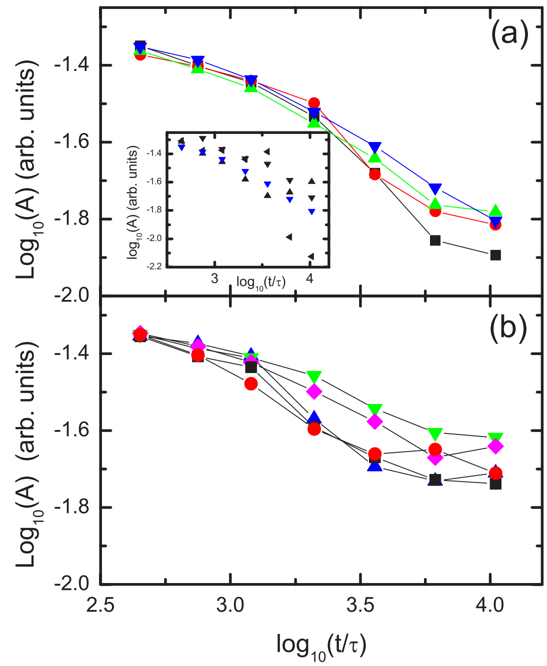

FIG. 4. (Color online) Both plots give boundary area (normalized by the total area of the image) as a function of time normalized by the director relaxation time. (a) is for noise strengths $\eta=0$ (■), 0.019 (red ●), 0.037 (green ▲), 0.056 (blue ▼). Inset in (a) illustrates the variation in individual runs of the normalized boundary area. Three different runs (indicated by the black symbols) are compared to the average for a noise strength of 0.056 (blue ▼). (b) is for noise strengths 0.074 (■), 0.093 (red ●), 0.148 (blue ▲), 0.167 (green ▼), and 0.185 (magenta ♦). There is a qualitative difference in the temporal evolution in the two regimes, $\eta<0.07$ and $\eta>0.07$ represented by (a) and (b), respectively, suggesting a transition in the temporal evolution as a function of the noise strength.

locations, formed in approximately 30 s. As these domains evolved, images were taken every 30 s for 35 min. At 35 min, there is generally only a small portion of a single domain wall within the field of view. For each noise amplitude, the analysis results for the 20 quenches were averaged. Noise amplitudes ranging from $\eta=0$ to $\eta=0.2$ were used.

In order to measure the degree of domain coarsening that has taken place, we look at the length of the boundaries between domains. The analysis was based on the algorithm described in Ref. [28], and the details, as applied to our system, are given in Ref. [10]. Essentially, one uses the fact that the product $k_x k_y$ for the wave vector of the zig pattern is positive, while for zag it is negative. The optical image is translated into a representation of the sign of $k_x k_y$, with regions of black and white corresponding to plus and minus, or zig and zag. The program then uses thresholding techniques to isolate the boundaries between domains that are mixtures of black and white pixels. The number of boundary pixels are normalized by the total number of pixels in the image, and this measure provides a method of quantifying the degree of domain coarsening at any given time. Figure 3 shows the typical results of the program for an image that is taken 6 min after the quench.

The main results of the paper are illustrated in Fig. 4. Here the time evolution of the average domain wall length is plotted for a number of different noise strengths. Time is measured from immediately after the quench. Also, time is scaled by the director relaxation time, $\tau \equiv \gamma_1 d^2/(\pi^2 K_{11}) = 0.2$ s. Here γ_1 is a rotational viscosity; K_{11} is the splay elastic constant; and d is the thickness of the cell. There are two main results. For comparison, three randomly selected individual runs are shown in the inset of Fig. 4(a) for one particular noise strength. This illustrates a typical variation between runs. (Note that the scale for the area axis is slightly increased in the inset to show the full range of the variations, but the time axis is the same scale as the full plot.)

Figure 4 illustrates that the evolution is slowed by increasing the noise strength. At a noise strength of 0.07, there is a qualitative change in the evolution of the system. (A noise strength of 0.07 corresponds to $\eta = 0.38$ V, which is well below the transition suggested for the onset voltage in Fig. 2.) Below 0.07, the system appears to continue to evolve on the time scale for which observation was possible. Above 0.07, the system plateaus at the the later times and the evolution is "frozen." Another way to view this is that the late time measure of domain wall length is significantly larger for quenches with a noise strength above 0.07, implying that the coarsening dynamics have slowed down. This is opposite to previous numerical results [2,11,12]. One likely explanation is the fact that we are using noise that is not purely multiplicative or additive. Another possibility is the role of the spatial character of the noise. These results suggest that a spatially random signal is necessary for increasing the coarsening rate, raising the interesting possibility that a deterministic temporal signal with a random spatial component might be sufficient for enhancing coarsening. Therefore, even though these experiments demonstrate one method for slowing the growth of domains, additional work is needed to fully explore all possible scenarios.

Another interesting result of applying this type of noise to the system is the global impact on the pattern. Under sufficiently strong amplitude, one would expect the noise to cause the spontaneous generation of dislocation pairs, resulting in some level of chaotic dynamics. We observed no evidence for such a transition, even at noise strengths as high as 0.9. This behavior has interesting implications for the nature of coupling between the applied noise and the pattern dynamics. The noise clearly impacted the dynamics of the system by significantly slowing domain growth at a critical value. However, the lack of any generation of defects is probably connected to the fact that the noise is applied directly to the voltage and that the driving force is given by the root mean square value of the voltage. The impact of this should be explored further with simulations and by experimental studies of different noise sources. It also presents an interesting pattern situation in which the dynamics of a system can be controlled to some degree by noise without having a substantial impact on the qualitative aspects of the pattern.

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